An Attempt for Description of Quasi–Autopoietic Systems Using Metabolism–Repair Systems

Tatsuya Nomura
Evolutionary Systems Department
ATR Human Information Processing Research Laboratories
2-2, Hikaridai, Seika-cho, Soraku-gun, Kyoto 619-02 JAPAN
Phone: +81–774–95–1010, Fax: +81–774–95–1008, E-mail: nomura@hip.atr.co.jp

Abstract

In this paper, we attempt to describe Autopoietic Systems using a mathematical framework of Metabolism–Repair Systems. Our description does not strictly reflect pure autopoiesis, and can only represent quasi–autopoietic systems. However, we believe that this attempt will spur discussions on the formal description of autopoiesis and the relation to mental and social systems, and so on, and we dare to propose a rough model.

1 Introduction

Autopoiesis is a neologism, introduced by Maturana and Varela to designate the organization of a minimal living system [8]. Maturana produced this theory from his research on visual nervous systems, and Varela developed his own system theory based on it. Later, Luhmann applied autopoiesis to the theory of social systems, developing his own interpretation of it [7].

Recently, this theory has been applied to not only sociology but also psychopathology [2]. Moreover, Kawamoto has continued his own development of autopoiesis. In his book, he designated the properties of autopoiesis by comparison with conventional system theories [6].

However, there is still no mathematical model that represents autopoiesis itself. A machine learning model inspired by autopoiesis was proposed to do tasks such as pattern recognition [3], but this model does not represent autopoiesis itself.

There are Metabolism–Repair Systems ((M, R) systems), a mathematical system model introduced by Rosen to formalize the functional activities of a living cell – metabolism, repair, and replication [9]. This system model maintains its metabolic activity through inputs from environments and repair activity.

Fontana and Buss suggested the similarity between their study and these systems [5]. In this paper, we try to describe autopoietic systems using a mathematical framework of (M, R) systems. Our attempt may be very thoughtless because of the difficulty in interpreting autopoiesis in the framework of the naive set theory. Accordingly, our description does not strictly reflect pure autopoiesis, and can only represent quasi–autopoietic systems. Nevertheless, we believe that this attempt will spur discussions on the formal description of autopoiesis and the relation between autopoiesis, mental systems, and social systems, and so on, which has been a highly controversial point. Based on this belief, we dare to propose a rough model of quasi–autopoietic systems.

We give an aspect of (M, R) systems in section 2, and an aspect of autopoiesis and a discussion on its interpretation in section 3. Then, we propose a description of quasi–autopoietic systems and explain a way of coupling them in section 4. In this paper, we focus our attention on cyclic systems and apply our description of quasi–autopoietic systems to systems of this type in section 5. Finally, we discuss the problems of our framework in section 6.

2 Metabolism–Repair Systems

(M, R) system is a model proposed as one solution to the question of how it is possible for the overall life time of an input–output system to be greater than that of any of its components. Rosen has stated the following [9]: Biological cells are continually repairing themselves. In order to keep the system functioning beyond the lifetimes of its components, it is necessary to replace components before their lifetimes has been exceeded.

The simplest (M, R) systems represent the above aspect in the following diagram:

$$A \xrightarrow{f} B \xrightarrow{\phi_f} H(A,B) \tag{1}$$

Here, A is a set of inputs from an environment to the system, B is a set of outputs from the system to the environment, f is a component of the system represented as a map from A to B, and ϕ_f is the repair component of f as a map from B to $H(A,B)$ ($H(X,Y)$ is the set of all maps from a set X to a set Y). In biological cells, f

corresponds to the metabolism, and ϕ_f to the repair. If $\phi_f(b) = f$ ($b = f(a)$) is satisfied for the input $a \in A$, we can say that the system self–maintains itself.

Moreover, a replicator component is introduced in the following diagram:

$$B \overset{\phi_f}{\to} H(A,B) \overset{\Phi_f}{\to} H(B, H(A,B)) \qquad (2)$$
$$(\Phi_f \in H(H(A,B), H(B, H(A,B))))$$

Here, the replicator component satisfies $\Phi_f(\phi_f(b)) = \Phi_f(f) = \phi_f$ for the above b and f. Figure 1(a) shows the simplest (M, R) system.

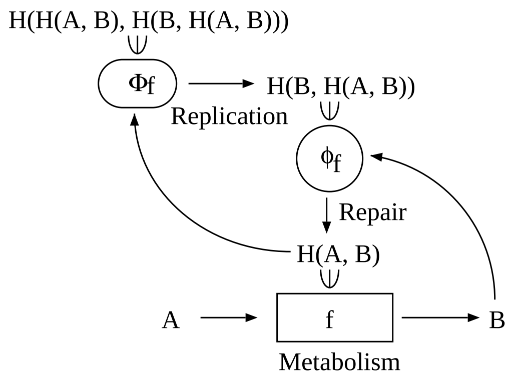
Diagram (2) also represents an (M, R) system. However, Φ_f can be constructed by the preceding (M, R) system in the following way:
For a and b such that $b = f(a)$ and $\phi_f(b) = f$, if $\hat{b} : H(B, H(A,B)) \to H(A,B)$ ($\hat{b}(\phi)(a') = \phi(b)(a')$ ($\phi \in H(B, H(A,B)), a' \in A$)) has the inverse map $\hat{b}^{-1}$, it is easily proved that $\hat{b}^{-1}(f) = \phi_f$. Thus, we can set $\Phi_f = \hat{b}^{-1}$.

General (M, R) systems are represented in the following form:

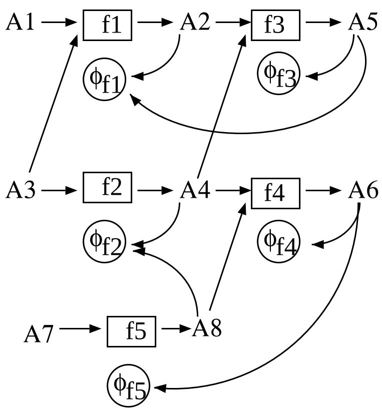
$$\begin{aligned}
&(\hat{A},\ \hat{F},\ \hat{A}_f,\ \phi_f) : \qquad (3)\\
&\hat{A} : \text{ a family of sets, } \hat{F} : \text{ a family of maps}\\
&{}^{\forall} f \in \hat{F}\ \ {}^{\exists} A_1, \ldots, A_m, B \in \hat{A}\ s.t.\ f \in H\left(\prod_{i=1}^{m} A_i, B\right)\\
&{}^{\forall} A \in \hat{A}\ \ {}^{\exists} f \in \hat{F}\ \ s.t.\\
&\quad A = \text{the domain of } f, \text{ or the range of } f\\
&{}^{\forall} f \in \hat{F}\ \ {}^{\exists} \hat{A}_f \subset \hat{A},\ \phi_f : \prod_{C \in \hat{A}_f} C \to H\left(\prod_{i=1}^{m} A_i, B\right)\\
&\quad \left(\prod_{i=1}^{m} A_i = \text{domain of } f,\ B = \text{range of } f\right)
\end{aligned}$$

Here, $A \in \hat{A}$ which are not any of the domains of $f \in \hat{F}$, are input sets from the outside of the system; those which are not any of the ranges of $f \in \hat{F}$ are output sets to the outside of the system. $\hat{F}$ is the set of the metabolism and ϕ_f corresponds to the repair for each $f \in \hat{F}$. Figure 1(b) shows general (M, R) systems. The repair for each metabolism depends on the output of itself or the others.

Casti developed a theory of (M, R) systems assumed as linear systems, to mathematically analyze the conditions for stability in self–maintenance [1]. In this paper, we borrow the framework of metabolism and repair in (M, R) systems and apply it to our description of the dynamical aspect of input–output behaviors, and the framework of replication and apply it to our description of the circular operations.



(a) Simplest (M, R) System



(b) General (M, R) System

Figure 1: (M, R) Systems

3 Autopoiesis

3.1 Aspect of Autopoiesis

Autopoiesis provides a framework in which a system exists as a living organization, based on physical and chemical processes. Maturana declared that living systems are machinery, and then claimed that autopoietic machinery is equivalent to living systems.

An autopoietic system is one that continuously produces the components that specify it, while at the same time realizing itself to be a concrete unity in space and time; this makes the network of production of components possible. An autopoietic system is organized as a network of processes of production of components, where these components:

1. continuously regenerate and realize the network that produces them, and

2. constitute the system as a distinguishable unity in the domain in which they exist.

Maturana gives a car as a representative example of a non–autopoietic system and claims the following [8]: The self–maintenance of a car as itself is realized only when there is a relation between inputs from a driver and outputs of the car. On the other hand, living systems self–maintains themselves by repeatedly reproducing the components and not by actions from others. Although they take nutritious substances from the outside, the organization is not determined corresponding to the substances. The processes for self–reproduction exist firstly and foremost, and the nutritious substances are subordinate to these processes.

The characteristics of autopoietic systems Maturana gives are as follows:

1. Autonomy:
   Autopoietic machinery integrates various changes into the maintenance of its organization. A car, the above example of a non–autopoietic system, does not have any autonomy.

2. Individuality:
   Autopoietic machinery has its identity independent of mutual actions between it and external observers, by repeatedly reproducing and maintaining the organization. The identity of a non–autopoietic system is dependent on external observers and such a system does not have any individuality.

3. Self–Determination of the Boundary of the System:
   Autopoietic machinery determines its boundary through the self–reproduction processes. Since the boundaries of non–autopoietic systems are determined by external observers, self–determination of the boundaries does not apply to them.

4. Absence of Input and Output in the System:
   Even if a stimulus independent of an autopoietic machine causes continuous changes in the machine, these changes are subordinate to the maintenance of the organization which specifies the machine. Thus, the relation between the stimulus and the changes lies in the area of observation, and not in the organization.

Moreover, Kawamoto positions dynamical stable systems which self–maintain themselves through metabolism to the outside, self–organizing systems such as crystals which grow while morphing themselves according to their environment, and autopoietic systems, as the first, the second, and the third generation systems, respectively [6]. Kawamoto particularly focuses on the fourth item among the above characteristics of autopoiesis, i.e., absence of input and output in the system.

Important is the view where the system is understood based on the production processes when we consider the "absence of input and output". Kawamoto claims the following: The view of the relation between inputs and outputs in the system is one from external observers and it does not clarify the organization or the operation of the production in the system. A living cell only reproduces its components and does not produce the components while adjusting itself according to the relation between itself and oxygen in the air. Although the density of oxygen affects the production processes, external observers decide the influence and not the cell. As long as the system is grasped from an internal view of the cell, the system does not have any "inputs and outputs".

The gist in the concept of autopoietic systems Kawamoto gives involves the following:

1. The set of components of a system is determined by the operation of the system.

2. The operation of the system precedes the initial condition.

3. The operation of the system is executed only to succeed itself and does not aim to produce by–products.

4. In the operation of the system, the things that happen in the system clearly differ from the things that external observers discriminate.

3.2 Difficulty in Interpretation for Autopoiesis

How systems are grasped from the view of external observers is interpreted as separating the observers from the environment including the system, distinguishing between the system and the background in the environment, and verifying the relation between the system and the distinguished background, that is, the outside of the system. Autopoiesis forces us to give up this view, that is, to put our view in the system, not in the outside of the environment.

Kawamoto gives the following statement as an example of this shift of view: If a person is fast running on the ground like drawing a circle, the person just continuously reproduces the action of running, although an external observer decides that the person is determining the boundary of the system. When the person stops running, the boundary vanishes.

However, this shift of view is not easily acceptable in the contemporary situation where the view of external observers is still major in natural science. If a person bounded to this view observes an autopoietic system, the view shifts towards the outside of the environment and the system is grasped as a static map or dynamical system in a state space. Even if the view shifts towards the inside of the system, the production processes of the components themselves are grasped as the object of the observation and the view of external observers is not completely given up. In the above example of a running person, the observer produces an image of the relation between the person as the object of the observation and the space where the person is running.

Moreover, as long as the view of external observers is not given up, the above gist of Kawamoto, in particular, the determination of the set of components by the operation and the precedability of the operation with the initial condition in the system cannot be understood. In the conventional system theory, state spaces where the operation is defined firstly exist, the initial condition is determined independent of the operation, and the properties in the state spaces by the operation such as time evolution are discussed.

A person bounded to the view of external observers cannot imagine the situation where the operation determines its domain and initial condition. Thus, such a person can imagine just self–organizing systems such as hyper–circles, which belong to the second generation systems Kawamoto claims.

It cannot be denied that we are bounded to the view of external observers. Yet, we attempt to verify whether there is a form of systems which satisfies the above gist of Kawamoto within the view of external observers.

4 Quasi–Autopoietic Systems

4.1 Description of Quasi–Autopoietic Systems

We cannot describe pure autopoiesis unless we give up the view of external observers. Thus, it is difficult to represent autopoiesis in the framework of the naive set theory, in which no operation is defined unless the domain and range set are defined in advance.

Here, we propose a description of systems which satisfies the gist of Kawamoto for autopoiesis in section 3 within the framework of the naive set theory; the system are called quasi–autopoietic systems. The underlying motivation for quasi–autopoietic systems is to verify whether it is completely impossible to introduce the characteristics of autopoiesis into conventional system theories and represent them as a computational model.

The most basic description of quasi–autopoietic systems is given in the following diagram:

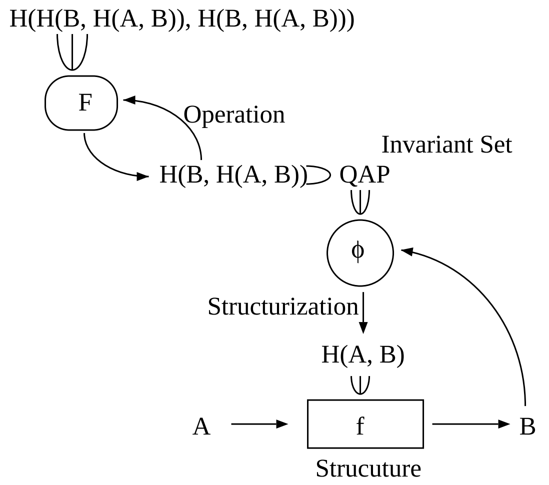
$$A \xrightarrow{f} B \xrightarrow{\phi} H(A,B) \quad (4)$$

$$H(B,H(A,B)) \xrightarrow{F} H(B,H(A,B)) \quad (5)$$

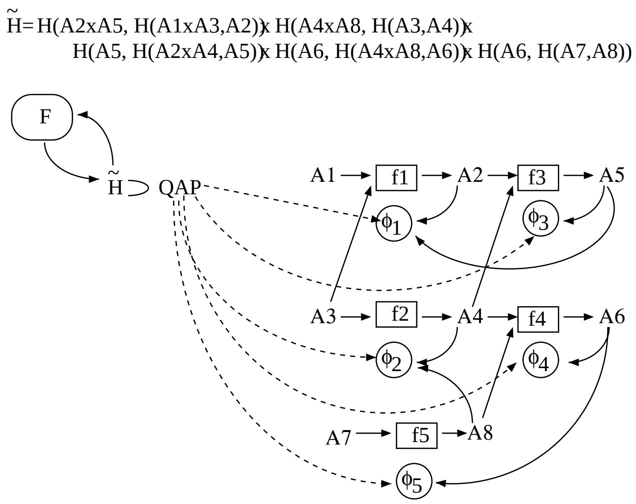
Here, (4) is the same form as (1) which represents the (M, R) systems in section 2. Instead of the replication map from $H(A,B)$ to $H(B,H(A,B))$ in the (M, R) systems shown in (2), the map F on $H(B,H(A,B))$ is given. This map determines the system's self by defining its self as an invariant set with a kind of ergodicity property for the map; that is, its self QAP is defined as follows:

$$QAP \subset H(B,H(A,B)),\ F(QAP) = QAP, \quad (6)$$

$$^{\forall}\phi, \phi' \in QAP, \quad ^{\exists}n \in \boldsymbol{N}\ s.t.\ F^n(\phi') = \phi \quad (7)$$



(a) The Most Basic Quasi–Autopoietic System



(b) General Quasi–Autopoietic System

Figure 2: Aspect of Quasi–Autopoietic Systems

Figure 2(a) shows an aspect of the most basic quasi–autopoietic systems. The system acts in the following way. The initial state of the system in $H(B,H(A,B))$ is selected within its self $QAP \subset H(B,H(A,B))$, and that in $H(A,B)$ is randomly selected. Next, given an input $a \in A$, the next states f' and ϕ' are determined in the following way:

$$b = f(a), \quad (a \in A,\ b \in B)$$

$$f' = \phi(b) = \phi(f(a)), \quad (f' \in H(A,B)) \quad (8)$$

$$\phi' = F(\phi) \quad (\phi' \in H(B,H(A,B))) \quad (9)$$

Because an (M, R) system's self is the metabolism f and the repair ϕ_f, the main point is to keep f unchanged through self–maintenance by ϕ_f. In the quasi–autopoietic system, f and ϕ are permitted to change within QAP because the system's self that is maintained is QAP. The essential action in the system is not the processing from the input set A to the output set B, but the iteration of F on $H(B, H(A, B))$.

Here, we verify whether this quasi–autopoietic system satisfies the gist of Kawamoto in section 3 by regarding the map F as the operation and ϕ as the components of the system:

1. QAP is an invariant set of F. Moreover, whether an element in $H(B, H(A, B))$ belongs to QAP is determined by F, that is, by the reachability from the initial state of the system through F, because of the ergodicity property in (7). Thus, we can interpret this aspect as the one satisfying the first point in the gist.

2. In order for the system to exist as a quasi–autopoietic system, the initial state must belong to an invariant set for F. Thus, we can regard this aspect as the one satisfying the second point in the gist.

3. The thing the system executes results in a new state being made within QAP by iterating F, and not in the input–output map f being produced. Thus, we can regard this aspect as the one satisfying the third point in the gist.

4. If external observers watch only the input–output relation in the system and the changes in the relation, the thing that happens in the system, that is, the iteration of F within QAP, differs from the things which these observers discriminate in the system, that is, f. Thus, we can regard this aspect as the one satisfying the fourth point in the gist.

In the same way as with (M, R) systems, general quasi–autopoietic systems are represented in the following form:

$$(\hat{A},\ \hat{H},\ \hat{A}_H, F): \tag{10}$$
$$\hat{A}: \text{ a family of sets, } \hat{H}: \text{ a family of sets of maps}$$
$$^{\forall}H \in \hat{H}\ \ ^{\exists}A_1, \ldots, A_n, B_1, \ldots, B_m \in \hat{A}\ \ s.t.$$
$$H = H\left(\prod_{i=1}^{m} A_i, \prod_{j=1}^{m} B_j\right)$$
$$^{\forall}A \in \hat{A}\ \ ^{\exists}H \in \hat{H}\ \ s.t.$$
$$A = \text{the domain in } H, \text{ or the range in } H$$
$$^{\forall}H \in \hat{H}\ \ ^{\exists}\hat{A}_H \subset \hat{A}$$
$$F: \tilde{H} \to \tilde{H} \quad \left(\tilde{H} := \prod_{H \in \hat{H}} H\left(\prod_{C \in A_H} C, H\right)\right)$$

Figure 2(b) shows an aspect of general quasi–autopoietic systems. In this case, the system's self QAP is an invariant set of $\tilde{H}$ with the ergodicity property for F. F represents the network between the components of the system.

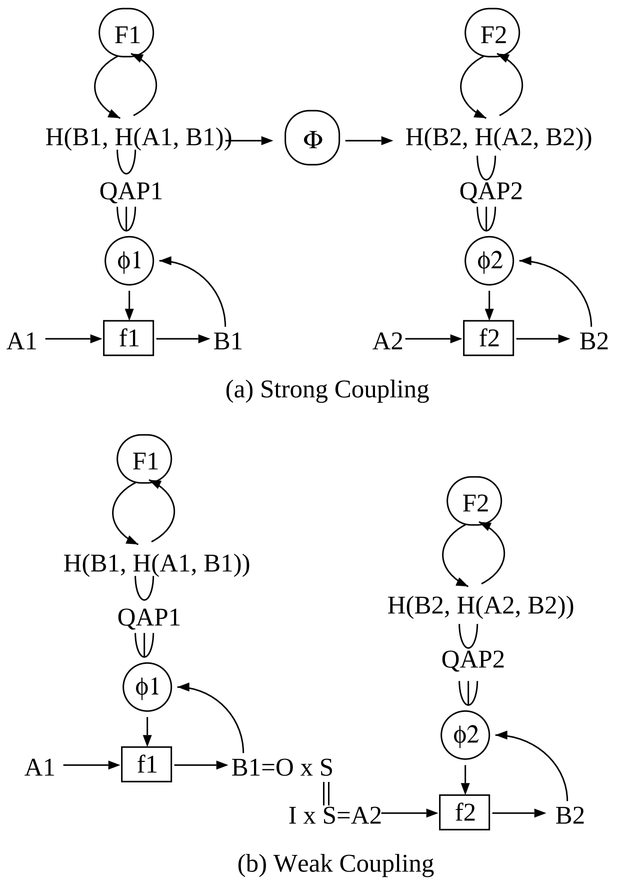


Figure 3: Coupling of Quasi–Autopoietic Systems

4.2 *Coupling of Quasi–Autopoietic Systems*

Using the quasi–autopoietic system in section 4.1, we can present a description for coupling them. In this paper, we give two kinds of couplings between quasi–autopoietic systems, called strong coupling and weak coupling.

In strong coupling, for two given quasi–autopoietic systems

$$\left.\begin{array}{l} A_i \overset{f_i}{\to} B_i \overset{\phi_i}{\to} H(A_i, B_i) \\ H(B_i, H(A_i, B_i)) \overset{F_i}{\to} H(B_i, H(A_i, B_i)) \\ QAP_i \subset H(B_i, H(A_i, B_i)) \end{array}\right\} \tag{11}$$
$$(i = 1, 2)$$

we assume that a map Φ from $H(B_1, H(A_1, B_1))$ to $H(B_2, H(A_2, B_2))$ exists such that the following commutative diagram is satisfied:

$$\begin{array}{ccc} H(B_1, H(A_1, B_1)) & \overset{F_1}{\to} & H(B_1, H(A_1, B_1)) \\ \downarrow \Phi & & \downarrow \Phi \\ H(B_2, H(A_2, B_2)) & \overset{F_2}{\to} & H(B_2, H(A_2, B_2)) \end{array} \tag{12}$$

Figure 3(a) shows the form of strong coupling. It is easily proved from the above diagram that $\Phi(QAP_1)$ is an invariant set with the ergodicity property in (7) for F_2. We define that these quasi–autopoietic systems are strongly coupled if $QAP_2 = \Phi(QAP_1)$. In this coupling, the former system determines the latter system. If the inverse map of ϕ exists, both systems mutually specify each other.

In weak coupling, for the given two quasi–autopoietic systems in (11), we assume the following situation:

$$B_1 \;=\; O \times S,\ A_2 = I \times S \qquad (13)$$

that is, a part of the output set of the former system is a part of the input set of the latter system. The latter system acts based on the output from the former system, which is a part of the environment of the latter system. Figure 3(b) shows the form of weak coupling. In this coupling, either the former or the latter system's self is not affected by the other.

5 Cyclic Systems based on Quasi–Autopoietic Systems

In autopoiesis and self–organized systems, a cyclic property is frequently discussed. In order to clarify the concept of quasi–autopoietic systems, we focus our attention on a particular form of systems, cyclic systems. In this paper, we verify the possible form based on the framework of quasi–autopoietic systems.

Now, we consider the following cyclic system:

$$A_i = I_i \times S_i,\ B_i = O_i \times S_{i+1} \qquad (14)$$

$$A_i \xrightarrow{f_i} B_i \xrightarrow{\phi_i} H(A_i, B_i) \qquad (15)$$

$$(i = 1, \ldots, n,\ S_{n+1} = S_1)$$

In this system, I_i is the set of the i–th inputs from the outside of the cycle, O_i is the set of the i–th outputs to the outside of the cycle, and they construct parts of the input and output sets of the i–th input–output unit. Moreover, another part of the output set of the i–th input–output unit is another part of the input set of the $(i+1)$–th unit. This cyclic system has a kind of auto–catalytic structure, e.g., a hyper–circle.

The cycle acts in the following way. The initial states of the cycle in $H(A_i, B_i)$ (f_i) and those in S_i (s_i) are randomly selected. Next, given an input $x_i \in I_i$, the next states f'_i and s'_i are determined in the following way:

$$(y_i, s'_{i+1}) \;=\; f_i(x_i, s_i) \qquad (16)$$

$$f'_i \;=\; \phi_i(y_i, s'_{i+1}) = \phi_i(f_i(x_i, s_i)) \qquad (17)$$

$$(y_i, s'_{i+1}) \in O_i \times S_{i+1},$$
$$(x_i, s_i) \in I_i \times S_i$$
$$(i = 1, \ldots, n,\ s'_{n+1} = s'_1)$$

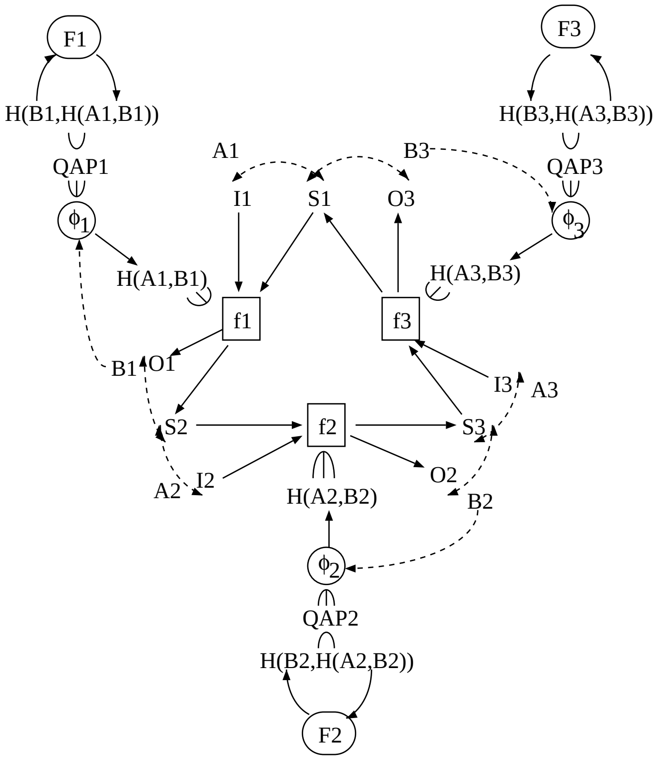


Figure 4: Weakly Coupled Cycle of Quasi–Autopoietic Systems ($n = 3$)

5.1 *Coupled Cycles*

In the form of the modification for ϕ_i, we can consider three kinds of cycles based on the framework of quasi–autopoietic systems: (1) weakly coupled cycle of quasi–autopoietic systems, (2) strongly coupled cycle of quasi–autopoietic systems, and (3) quasi–autopoietic cycle.

In weakly coupled cycles, the most basic quasi–autopoietic systems are coupled cyclicly and weakly as shown in section 4.2. The modification of ϕ_i to ϕ'_i is done by the following form:

$$H(B_i, H(A_i, B_i)) \xrightarrow{F_i} H(B_i, H(A_i, B_i)) \qquad (18)$$

$$\phi'_i = F_i(\phi_i) \qquad (19)$$

$$(i = 1, \ldots, n)$$

As shown in (15) and (18), the i–th unit makes a quasi–autopoietic system and it is weakly coupled with the $(i+1)$–th quasi–autopoietic system, cyclicly. Figure 4 shows an aspect of the weakly coupled cycle. In this cycle, each quasi–autopoietic system has an independent self QAP_i through F_i and it does not affect the others.

In strongly coupled cycles, the most basic quasi–autopoietic systems are coupled cyclicly and strongly as shown in section 4.2. Since the i–th unit makes a quasi–autopoietic system and it is cyclicly coupled with the

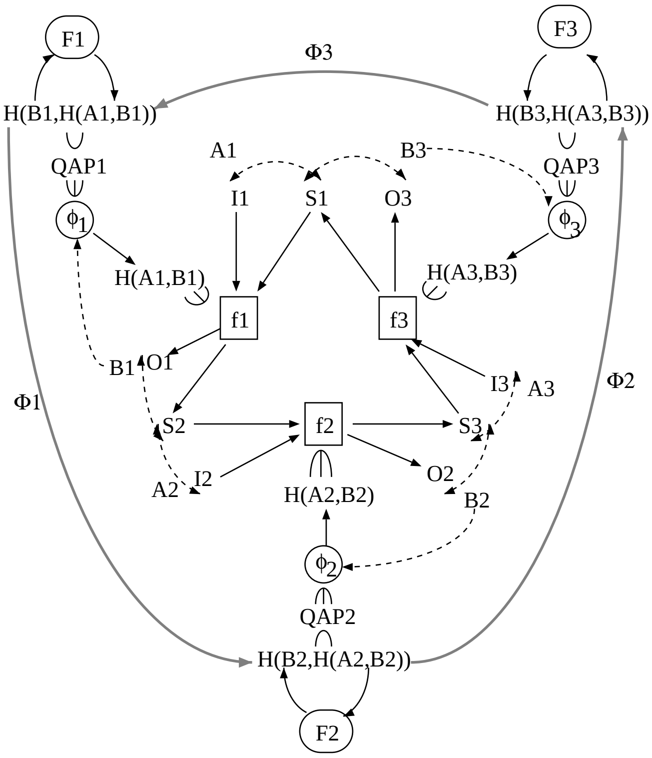


Figure 5: Strongly Coupled Cycle of Quasi–Autopoietic Systems ($n = 3$)

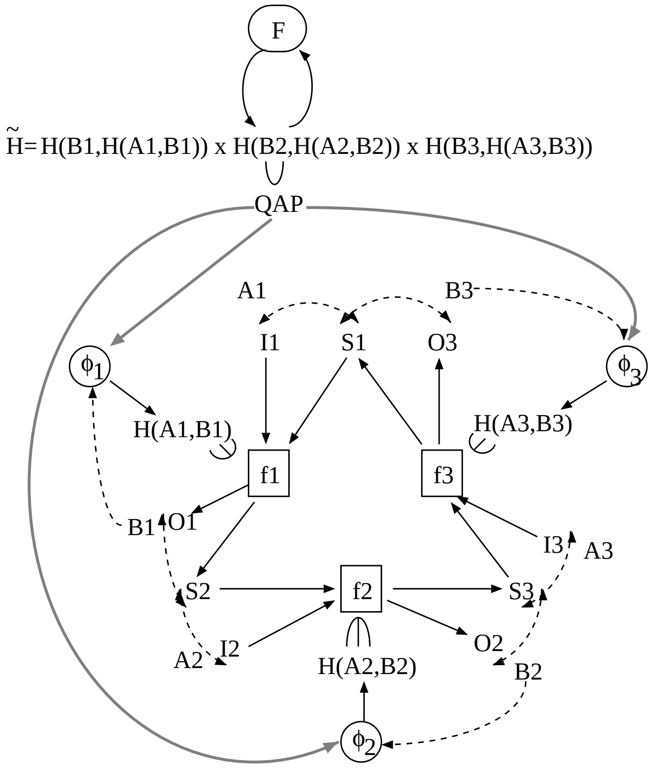


Figure 6: Quasi–Autopoietic Cycle ($n = 3$)

$(i+1)$–th quasi–autopoietic system in the same way as for the weakly coupled cycle, the modification of ϕ_i to ϕ'_i is done by the same form as (18) and (19). However, the modification of ϕ_i is done while maintaining the following boundary condition for the maps $\{\Phi_i\}_{i=1}^n$ which are associated with strong coupling:

$$\begin{array}{ccc} H_i & \overset{F_i}{\rightarrow} & H_i \\ \downarrow \Phi_i & & \downarrow \Phi_i \\ H_{i+1} & \overset{F_{i+1}}{\rightarrow} & H_{i+1} \end{array} \tag{20}$$

$$\phi_{i+1} = \Phi_i(\phi_i) \tag{21}$$

$$\left(\begin{array}{l} i = 1, \ldots, n,\ H_i = H(B_i, H(A_i, B_i)) \\ A_{n+1} = A_1,\ B_{n+1} = B_1, \\ F_{n+1} = F_1, \phi_{n+1} = \phi_1 \end{array} \right)$$

By the commutative diagram in (20), this boundary condition is maintained through the operation if the initial state of the cycle in H_i satisfies it. Figure 5 shows an aspect of the strongly coupled cycle. In this cycle, although each quasi–autopoietic system has its own self QAP_i through F_i, it does affect the others through the strong coupling by Φ_i.

Quasi–autopoietic cycles are not a system which can be represented as a coupling of quasi–autopoietic systems, but one general quasi–autopoietic system as shown in section 4.1. Thus, each input–output unit does not correspond to a quasi–autopoietic system, and the cycle has its own self QAP through the map F. The modification of ϕ_i to ϕ'_i is done in the following form:

$$F \quad : \quad \tilde{H} \rightarrow \tilde{H} \tag{22}$$

$$\left(\tilde{H} = \prod_{i=1}^{n} H(B_i, H(A_i, B_i)) \right)$$

$$(\phi'_1, \ldots, \phi'_n) = F(\phi_1, \ldots, \phi_n) \tag{23}$$

Figure 6 shows an aspect of the quasi–autopoietic cycle. Although several systems' selves can exist in coupled cycles, this cycle's self is not divided into subsystems.

5.2 Coupling of Cycles

Based on the forms of cycles shown in the above sections, we can extend the coupling of quasi–autopoietic systems to the coupling of coupled cycles or quasi–autopoietic cycles. Here, we can consider two types of couplings, weak coupling and strong coupling in the same way as section 4.2.

In the weak coupling of cycles, the quasi–autopoietic system in a coupled cycle or the general quasi–autopoietic system corresponding to a quasi–autopoietic cycle is weakly coupled with another. In other words, for a cycle shown in (14) and (15) and another cycle in the following:

$$C_j = P_j \times T_j,\ D_j = Q_j \times T_{j+1} \tag{24}$$

$$C_j \overset{g_j}{\to} D_j \overset{\varphi_j}{\to} H(C_j, D_j) \tag{25}$$

$$(j = 1, \ldots, m,\ T_{m+1} = T_1)$$

the i–th input–output unit in the former cycle and the j–th unit in the latter cycle are coupled through $O_i = P_j$. Figure 7 shows a weak coupling of cycles. This coupling is not dependent on the form of cycles shown in the above sections, but only on the level of input–output units.

In the strong coupling of cycles, the quasi–autopoietic system in a coupled cycle or the general quasi–autopoietic system corresponding to a quasi–autopoietic cycle is strongly coupled with another. In other words, the most basic quasi–autopoietic system shown in (18) of a coupled cycle or the general quasi–autopoietic system shown in (22) of a quasi–autopoietic cycle is strongly coupled with the most basic quasi–autopoietic system of another coupled cycle

$$H(D_j, H(C_j, D_j)) \overset{G_j}{\to} H(D_j, H(C_j, D_j)) \tag{26}$$

$$(j = 1, \ldots, m)$$

or the general quasi–autopoietic system of another quasi–autopoietic cycle

$$G \quad : \quad \tilde{W} \to \tilde{W} \qquad \left(\tilde{W} = \prod_{j=1}^{m} H(D_j, H(C_j, D_j)) \right) \tag{27}$$

Figure 8 shows a strong coupling of cycles. This coupling is dependent on the form of cycles, and three forms of couplings are considered: (a) coupling of coupled cycles of quasi–autopoietic systems, (b) coupling of a coupled cycle and a quasi–autopoietic cycle, and (c) coupling of quasi–autopoietic cycles. In case (a), the i–th quasi–autopoietic system's self QAP_i in a coupled cycle specifies the j–th system's self QAP'_j in another cycle. In case (b), a quasi–autopoietic cycle's self QAP specifies the quasi–autopoietic system's self QAP'_j in a coupled cycle, or the opposite specification is done. In case (c) a quasi–autopoietic cycle's self QAP specifies that of another quasi–autopoietic cycle QAP'.

6 Discussion

Through the previous sections, we gave a description for quasi–autopoietic systems using the framework of (M, R) systems, coupling of them, and the representation of cyclic systems based on the description. Although this is an attempt to possibly represent autopoiesis within the naive set theory, many problems remain.

First, we introduced an invariant set with the ergodicity property in the set of repair maps in (M, R) systems to represent the self–determination of the system's

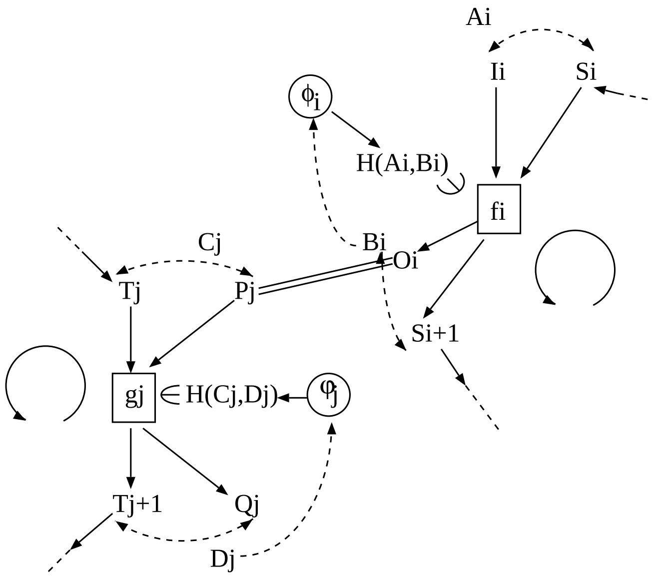


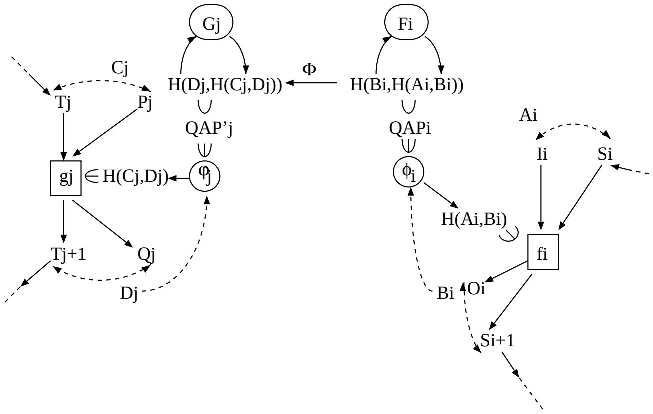
Figure 7: Weak Coupling of Cycles

boundary. However, this is only to give a class for an equivalent relation on the set given in advance by the operator given in advance. In other words, the space where the system's self exists is determined in advance and then the operator is also determined as the map on it. In pure autopoiesis, the set of components must be determined by the operator in the system, that is, the domain and range of the operator must be determined through the operator itself.

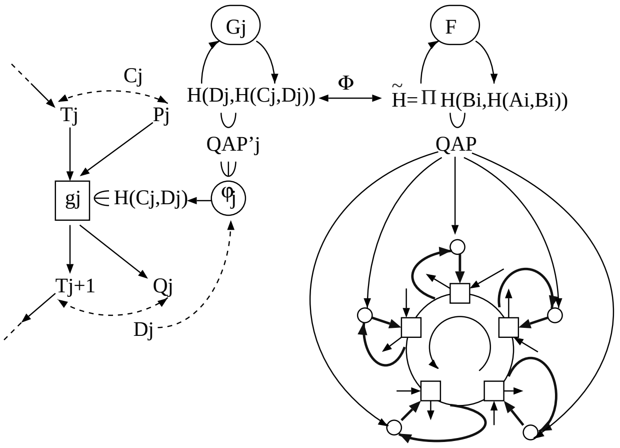
Second, we separated the operator and input–output actions in the system to represent the absence of inputs–outputs in the system. This caused the input–output actions to not affect the self–determination of the system's boundary and the operator acted independent of the outside of the system. If we regard the input–output actions as a part of the components, however, what happened was not realistic. Although we introduced coupling of the systems, this is a very provisional idea. Thus, our description does not strictly reflect the properties of pure autopoiesis, and can represent just quasi–systems as mentioned in section 1.

As a solution for these problems, we consider the effectiveness of λ–calculus for the representation of the interaction between the components and operator inspired by the approach which Fontana and Buss have proposed [4], because it can deal with the components and operator in the same level.

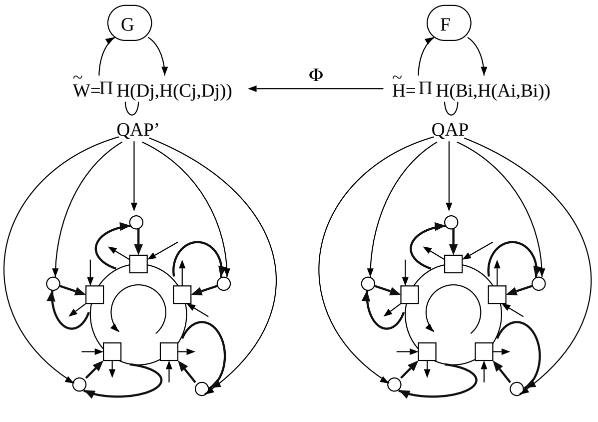
Even if our description is narrowly meaningful, there are many problems to be solved. First, we have not done either a comparison of our description with conventional system theories or a verification of the adequacy to the representation of concrete systems such as living cells,



(a) Coupling of Coupled Cycles



(b) Coupling of a Coupled Cycle and a Quasi–Autopoietic Cycle



(c) Coupling of Quasi–Autopoietic Cycles

Figure 8: Strong Coupling of Cycles

mental systems, and social systems, yet. This is difficult, but must be done as quickly as possible if we aim to propose a new system theory.

Moreover, we have to clarify mathematical properties within our description, construct a concrete model on a computer, and execute simulations to verify the properties, because our purpose is to represent the characteristics of autopoiesis as a computational model.

These must be solved as future problems.

Acknowledgment

The author would like to thank Dr. Katsunori Shimohara at ATR Human Information Processing Research Laboratories for his support.

References

[1] J. L. Casti. The Theory of Metabolism–Repair Systems. *APPLIED MATHEMATICS AND COMPUTATION*, 28:113–154, 1988.

[2] L. Ciompi. *Affektlogik – Über die Struktur der Psyche und ihre Entwicklung.* Ein Beitrage zur Schizophreneiforschung, 2. Auflage, Klett–cotta, Stuttgart, 1982 (Japanese Edition: Gakuju Shoin, Publishers, Ltd., 1994).

[3] G. Deffuant, T. Fuhs, E. Monneret, P. Nourgine, and F. Varela. Semi–Algebraic Networks: An Attempt to Design Gemometric Autopoietic Models. *Artificial Life*, 2(2):157–177, 1995.

[4] W. Fontana and L. W. Buss. "The Arrival of the Fittest": Toward a Theory of Biological Organization. *Bulletin of Mathmatical Biology*, 56:1–64, 1994.

[5] W. Fontana and L. W. Buss. The barrier of objects: From dynamical systems to bounded organizations. In J. Casti and A. Karlqvist, editors, *Boundaries and Barriers*, pages 56–116. Addison–Wesley, 1996.

[6] H. Kawamoto. *Autopoiesis: The Third Generation System.* Seido–sha Publishers, 1995 (Japanese).

[7] G. Kneer and A. Nassehi. *Niklas Luhmanns Theorie Sozialer Systeme.* Wilhelm Fink Verlag, 1993 (Japanese Edition: Shinsen–sha Publishers, 1995).

[8] H. R. Maturana and F. J. Varela. *AUTOPOIESIS and COGNITION: THE REALIZATION OF THE LIVING.* D. Reidel Publishing Company, 1980 (Japanese Edition: Kokubun–sha Publishers, 1991).

[9] R. Rosen. *FOUNDATIONS OF MATHEMATICAL BIOLOGY*, Vol. 2, chapter 4. Some Relational Cell Models: The Metabolism–Repair Systems, pp. 217–253. Academic Press, 1972.